THE DIFFERENT MATHEMATICAL APPROXIMATION FORMATS FOR THE FIELD EMISSION SPECIAL MATHEMATICAL FUNCTION v(*x*)

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ABSTRACT

Within the context of smooth-planar-metal-emitter (SPME) theoretical methodology, the best simple model for describing the field electron emission (FE) tunnelling barrier is the Schottky-Nordheim (SN) ("planar image rounded") barrier. Use of this barrier within the context of the simple (or "first-order") JWKB mathematical tunnelling formalism leads to so-called Murphy-Good-type FE equations. For experiment-facing theory, the author now prefers to use a so-called *Extended Murphy-Good (EMG) FE Equation* to describe the relationship between the emission current I_e and the emitter's characteristic local barrier field F_C and local work function ϕ (usually taken as the emitter-apex values). This EMG equation has the form

$$I_e^{EMG} = A_f^{SN}\, a\phi^{-1} F_C^{\,2} \exp[-v_F b\phi^{3/2}/F_C],$$

where a and b are the Fowler-Nordheim constants, A_f^{SN} is the formal emission area for the SN barrier and v_F is a particular value (appropriate to a barrier described by ϕ and F_C) of the principal field emission special mathematical function v(x). Here, v(x) is expressed as a function of the *Gauss variable x*, which is my name for the independent variable in the Gauss Hypergeometric Differential Equation. The function v(x) is a special solution of this equation, and it can be shown that v(x) is applied to FE theory by setting $v_F = v(x = f_C)$, where the *characteristic scaled field* f_C is related to the barrier field F_C by $f_C \equiv (e^3/4\pi\varepsilon_0)\phi^{-2}F_C$.

We now know that v(x) is an unusual mathematical function in that its efficient representation as an exact series expansion requires TWO infinite power series, rather than one. Hence, efficient representation CANNOT be obtained by simple Taylor expansion.

In FE literature there exist many (around 20) different approximate mathematical formulae for v_F, mostly expressed as functions of f_C, or more commonly of the related variable y [$=+\sqrt{f_C}$] called the "Nordheim parameter". All of these can be seen as derived from different mathematical approximations for v(x). The purpose of this Poster is to classify these different approximations for v(x), and indicate typical mathematical accuracies in the range $0.15 \leq x \leq 0.45$, which is the "pass" range for f_C in the orthodoxy test [see R.G. Forbes, Proc. R. Soc. Lond. A **469**, 20130171 (2013)].

The main divisions of the classification are: (1) whether it is an "old" approximation, effectively based on a single power series, or a "new" approximation, effectively based on the (mathematically correct) use of two power series; (2) how many terms are used in the approximation.

It will be pointed out that the use of "old" approximations (which is still widespread in current literature) should now be regarded as obsolete, first because the "new" approximations are of higher accuracy, second because the "old" approximations do not lead to "best physics" in the analysis of experimental current-voltage data or in the on-going development of improved data-analysis theory.